Level of Rationality Shifting: Evidence from Experimental Data

Term Paper for ECO421F: Topics in Experimental Economics

Tianyu Du*

Instructor: Yoram Halevy

December 13, 2019

Contents

*Student Number: 1003801647; E-mail: `tianyu.du@mail.utoronto.ca`.

1 Introduction

Classical microeconomic theories have heavily relied on strict assumptions on agents' rationality. Undeniably, these assumptions are promising in most cases and allow most economic theories to be justified using elegant mathematical frameworks. In mainstream game-theoretical models, agents are often assumed to be completely rational and able to conduct infinitely many rounds of iterative eliminations at zero cognitive cost. Moreover, every player is assumed to possess the belief that all other players are rational as well. In scenarios where the wisdom of the crowd plays a significant role in decision making, such rationality assumption appears to be well-justified. For instance large tech companies with a group of market experts may behave rationally: while analyzing tech companies' decisions on when to announce new products, the sequential-entry game has been shown to be predictive. However, studies on individual decision makings have provided evidences showing agents are not following game-theoretical reasoning most time.

Game-theoretical models assume that players are able to reason at an infinite depth while assuming perfect rationality of other players. This assumption implicitly rules out the possibility that individuals may behave differently given different beliefs on opponents' intelligence. In the experiment setting discussed in this paper, subjects were undergraduate students from the University of Toronto, and each of them was playing against both undergraduate and graduate students. Following the assumption proposed by classical theories, subjects would behave identically as they should believe both undergraduate and graduate students to be perfectly rational and possess equal cognitive capability. Results from our study have shown a discrepancy between subjects' behaviours while facing different opponents.

Understood that most individuals do not make decisions based on complete rationality, and their responses are highly dependent on other players, the significance of understanding how they behave becomes salient.

2 Research Question

Understood that different individuals may have different perceptions regarding their opponents, and would behave accordingly, it is natural to explore the discrepancy in different scenarios with finer details. The central question of this study is twofold: *(i) whether individuals adapt their behaviours significantly while facing more sophisticated opponents? (ii) how do they modify their behaviours based on their beliefs on their opponents?*

The first question can be framed into an *identification* problem trying to recover different behaviour patterns and analyze whether the relationship between the behaviour discrepancy and the sophistication of opponents is statistically significant. Meanwhile, the second question can be structured into a *pattern recognition* problem, in which one wishes to explore the systematic differences in behavioural patterns revealed through experiments. Moreover, thanks to the nature of experimental design, reverse causality can be eliminated immediately, if any correlation between behavioural pattern and players' belief of opponents is observed from the dataset, one may conclude the relationship to be causal directly.

In the following sections, this paper is going to address both the identification problem and the recognition problem, while emphasizing the pattern recognition part more. Specifically, a short literature review is presented in the next section. To elaborate on experimental design and observations more concisely, several definitions are covered afterward. Then the main experimental designs are reviewed and purposes of specific designs are explained. The subsequent section is devoted to experimental data and relationships detected from the data. Next, two central questions mentioned above are revisited to check whether these data collected helped answer them, after which discussions and concluding remarks follow.

3 Literature Review

In a long period of time, economic theories are built on purely rational agents. For instance, theories of general equilibrium are built on the collective behaviour of rational and selfish agents. The development of game theory over the past half-century relaxed the collective nature of agents a little bit: individuals do not have to act collaboratively and flawlessly any more, complex interactions among entities, which has been ignored for a long time, were carefully examined. However, almost all theories are still based on rigorous rationality assumptions. What if individuals' rationalities are actually bounded, moreover, what if levels of rationality are actually different for individuals?

Albeit the success of theories on the aggregate level, the picture on an individual level is quite different. Time and cognitive constraints are more strict in this case, so that individuals, in general, fail to be entirely rational. Empirical results have shown that individual agents behave quite differently from a hypothetical oracle player does. The well-known "guessing $\frac{2}{3}$ of the average" game reveals that individual agents are in fact following the iterative reasoning path proposed by classical models, but only conduct limited rounds of iterative reasoning Nagel 1995. In particular, classical game-theoretical models suggest the only Nash equilibrium resulted from iterative elimination is every player plays zero. The original experiment results indicated that only an insignificant portion of subjects chose the action supported by rationality-based models. It turns out that either typical players are not completely rational or do not believe their opponents are rational. The discrepancy between predictions from rationality-based models and empirical observations has revealed the demand for new theories.

A later work proposed a framework to determine individuals' level of rationality using a cost benefit analysis framework (Alaoui and Penta 2016). Their framework explicitly modelled different cognitive costs of playing against opponents at various levels. Another different model-free approach has been taken as well (Kneeland 2015). Kneeland used an innovative ring game design to unravel subjects' levels of rationality. In later discussions in this paper, a mixture of these methodologies is pursued. In particular, we only impose weak assumptions on individuals' preferences, and identify individuals' levels from experimental data.

4 Model

4.1 Definitions and Games

Unlike in most economic studies, this study does not impose prior belief on how rational subjects are. Instead, analysis conducted here is designed to estimate how rational subjects are. That is, subjects were acting optimally in order to maximize the compensated payoff after the experiment. Participants were convinced that the final monetary payoff would be positively correlated with values presented in payoff matrix in each game, and the actual payment amount would be determined by one game chosen randomly from all games played. Therefore, it is reasonable to assume that subjects would try their best in each game.

The entire set of experiments consist of four games (one large 4x4 game and three small 3x3 games), and subjects were asked to play against two sets of players: undergraduate opponents and Ph.D opponents. For subject i, the session played against opponent p can be expressed as

$$g_{k,p} = \langle (i,p), (A_i, A_p), (\succsim_i, \succsim_p) \rangle \quad k \in \{L, S_1, S_2, S_3\},\ p \in \{ug, grad\} \tag{1}$$

Let $\mathcal{N}$ denote the pool of all subjects,

Definition 4.1. The **shallow belief** of one player $i \in \mathcal{N}$ refers to his/her understanding of the structure of game $\mathcal{G}_{k,p}$.

Definition 4.2. The **higher-order belief**(or simply **belief**) of one player $i \in N$ refers to his/her perception of opponent's belief, and how would the opponent behave accordingly.

Often, the subscript of game is omitted, in this case, g_p denotes a generic game played against opponent of time p. And g_k with $k \in \{L, S_1, S_2, S_3\}$ denotes the particular game.

4.2 Identifying Levels of Rationality

Let $\mathcal{G} = \{g_k : k \in \{L, S_1, S_2, S_3\}\}$ denote the set of game. Provided that a player i is playing using a certain reasoning denoted as π while facing certain type of opponent p (i.e., undergraduate or Ph.D.), the observed choices of player i should be rationalized by π in all games played against p.

The procedure of identifying level of rationality is independent from the type of opponent played against, hence, the notion of p is omitted in the following discussion.

Let $\mathcal{A}$ denote the action space. Then, a mapping $\phi : \mathcal{G} \times \mathcal{A} \rightrightarrows \mathbb{Z}_+$ is constructed such that $\phi(g, \theta)$ represents the collection of all levels that would choose action θ in game g. For instance, consider player Y's options in game S_1 (see Appendix A), player Y at level 1 would play a and player Y at level ≥ 2 should choose c. Note that it is possible for a level 0 player to choose any action, therefore, in this case,

$$\phi(g_{S_1}, a) = \{0, 1\} \tag{2}$$

$$\phi(g_{S_1}, b) = \{0\} \tag{3}$$

$$\phi(g_{S_1}, c) = \{0, 2, 3, \cdots\} \tag{4}$$

Definition 4.3. Given a game $g \in \mathcal{G}$, a subject i 's action $\theta_{i,g}$ can be **rationalized** by the level $k \in \mathcal{R}$ if $k \in \phi(g, \theta_{i,g})$.

Given that for each p, only four games were played, we don't have sufficient data (i.e., degree of freedom) to identify the level that is too high. For example, a level 20 player cannot be well-identify using data available. To compromise, the algorithm proposed here can only identify the level from a pre-defined $\mathcal{R} \subsetneq \mathbb{Z}_+$, typically, $\mathcal{R} = \{0, 1, 2, 3, 4, 5\}$.

Definition 4.4. The **hard level** of rationality of a player i is defined to be the greatest level $\ell_i^h \in \mathcal{R}$ that rationalizes this player's actions, $(\theta_{i,g})_{g \in \mathcal{G}}$. Specifically,

$$\ell_i^h := \max\{\ell \in \mathcal{R} : \ell \in \phi(g, \theta_{i,g})\ \forall g \in \mathcal{G}\} \tag{5}$$

It might be too harsh to require subjects to act according to exact level precisely in all games. To deal with this issue, we introduce another notion of soft level, which allows player to *make at most one mistake.*

Definition 4.5. The **soft level** of rationality of a player i is defined to the greatest level $\ell_i^s \in \mathcal{R}$ that rationalizes player's action in at least three out of the four games.

$$\ell_i^s := \max\{\ell \in \mathcal{R} : \ell \in \phi(g, \theta_{i,g}) \text{ for at least three } g \in \mathcal{G}\} \tag{6}$$

However, what if a player's actions in all three small games can be rationalized using level 4, but (s)he made a stupid mistake in the big game (say this player played c, which can only be rationalized by level 0), is this player still level 4? A great portion of subjects did this in the actual experiment, to incorporate this kind of pattern, a weaker definition of soft controlled level of rationality is introduced. This notion turned out to be a criterion groups subjects evenly (see data section).

Definition 4.6. A player i is said to have **soft controlled level** ℓ if player i is at soft level ℓ. Moreover, in the game (s)he made mistake in, g', his/her action must be rationalized by level $\ell - 1$. That is,

$$\forall g' \in \mathcal{G}\ s.t.\ \ell \notin \phi(g', \theta_{i,g'}),\ \ell - 1 \in \phi(g', \theta_{i,g'}) \tag{7}$$

Levels inferred from both definitions are examined in this paper. Moreover, as mentioned before, subjects may use the Nash reasoning instead of an iterative one. In particular,

Definition 4.7. A player i's (in the role of player Y) actions can be rationalized by a **Nash reasoning** if for every $g \in \mathcal{G}$, $\theta_{i,g}$ is in the support of Nash equilibrium strategy.

Even though all normal form games presented have unique Nash equilibria, not all of them possess a pure strategy Nash equilibrium. Specifically, only the first and third small games have pure strategy Nash equilibria. Indeed, we don't expect many subjects to understand the mechanics of deriving a mixed strategy Nash equilibrium, therefore, introducing another notion weaker than Nash reasoning becomes necessary.

Definition 4.8. Let $\mathcal{G}'$ denote the collection of games with pure strategy Nash equilibria. A player i's (in the role of player Y) actions can be rationalized by a **Nash reasoning** if for every $g \in \mathcal{G}'$, $\theta_{i,g}$ is in the Nash equilibrium strategy.

5 Experimental Design

Each experiment session includes two sections, the first section involves four games and aims to identify changes in subjects' choice while facing opponents with different backgrounds. Afterward, the second section is designed to elicit and measure individuals' valuations of each game played in the first section. Merging the result from two games helps to identify how subjects' perceptions of best responses and expected payoffs change with their perception of their counterparts.

5.1 Part I: Identifying the Behaviour Pattern

The first section aims to analyze behavior patterns using a comparative static manner. Specifically, for each of the four games in this section, subjects are instructed to make decisions twice in two hypothetical scenarios: when they are playing against another undergraduate student or against a Ph.D. student in economics. By reminding each subject that his/her counterpart is a Ph.D. student taking several graduate-level game theory courses, the instruction forces this subject to form a strong belief on his/her opponent's sophistication.

All games are presented in normal forms, only the first game is 4-by-4 and designed to capture an overarching pattern and the following three 3-by-3 games are used to deliver a more careful analysis on subjects' reasoning pattern. The different game sizes help determine whether subjects' responses depends on the complexity of game.

The first game has no pure strategy Nash equilibrium, but a unique mixed-strategy equilibrium does exist (see figure below). However, given the majority of subjects are undergraduate students, they are not supposed to identify and reason via a rigorous game-theoretical manner.

Y \ Z	A	B (2,3)	C (1,4)	D
a (1,2,5,NE)	14,0	12,24	14,12	0,0
b (3,4,NE)	2,10	24,0	0,16	12,4
c	22,10	0,2	4,0	8,22
d	12,10	8,10	0,12	10,8

Figure 1: 4x4 big game, underlined payoffs represent the best responses, numbers beside each action are levels rationalize the action, only positive levels are included, level 0 rationalizes all actions. NE denotes the action is in the support of Nash equilibrium strategy. Details on player Z are omitted.

It is worth mentioning that even though the first game does not have any pure strategy Nash equilibrium, players still can reason their opponents' behaviour using iterative elimination of never best responses and act accordingly. Players' levels of rationality and beliefs of the other player can be inferred from their choices.

In the following discussion, this essay primarily focuses on deducing subjects' behavioural patterns when they are in the role of player Y (vertical player with action a, b, c, d). However, games in this section are designed to be more or less symmetric, most arguments would be applicable to player Z (horizontal player with action A, B, C, D) after minor modifications.

As mentioned before, since any observed behaviours can be explained by saying the subject is choosing randomly (level 0), one cannot tell the exact level of rationality of the subject, but one may follow a hypothesis testing style reasoning. For instance, observing a certain player chose a clearly irrational action provides an upper bound of the subject's level of rationality, which quantify the set of possible levels this subject is at.

In particular, action d is a never-best-response (NBR) for player Y in the 4x4 game, choosing d would suggest the subject to be at most level 0. Similarly, action A is an unconditional NBR for player Z, with this knowledge in mind, if player 1 is at level one or higher, (s)he would figure this out and act accordingly.

To identify the possible levels of player Y, an iterative algorithm can be used. Given the common belief that level 0 player would choose his/her action randomly, a level 1 player best responses this by choosing the action carrying the greatest average payoff. In the 4x4 game, a level 1 player Y would choose a, and a level 1 player Z would choose C. Similarly, level 2 player Y, by definition, would take the unique best response to level 1 player Z's action by choosing a, and level 2 player Z is expected to choose B. This algorithm can be applied on all four games to construct the $\phi : \mathcal{G} \times \mathcal{A} \rightrightarrows \mathbb{Z}_+$ mapping mentioned in the previous section. For example, observing one subject's final choice to be b in the large game implies this subject is potentially level 0, 3, or 4.

In the first small game, even though the player Y does not have any unconditionally NBR, one can still conclude subject (as player 1) to be irrational (level 0) if the subject was choosing b. Similarly for an individual having no idea on his/her opponent's reasoning, (s)he is expected to choose a

because a carries the highest expected payoff conditioned on the belief that player Z acts following a uniform mixed strategy. Moreover, player Z has A as a strictly dominant strategy. Any subject in the role of player Y with a rationality of level 2 or higher is expected to choose action c.

Moreover, this game has a pure strategy Nash equilibrium, observing a subject choosing a or b helps to reject the hypothesis that the subject is following Nash reasoning.

Y \ Z	A(1+,NE)	B	C
a (1)	0,14	12,12	12,10
b	2,6	18,2	0,4
c (2+,NE)	12,10	0,8	8,8

Figure 2: 3x3 G1: the first small game

The second and third small games serve as another filter on subjects' levels of rationality. By combining observational data collected from this and previous games, one can infer subjects' levels and their beliefs with reasonable confidence. For instance, to conclude that one particular subject behaves as if (s)he is at level 1 while his/her counterpart is a Ph.D. student, we must observe this subject chose a in all four settings.

As mentioned, this criterion can be substituted using a soft version as well, one may allow the subject to "misbehave" at most once but still conclude him/her to be at a particular level. For example, if someone chose a in all small games, but took c in the large, it is possible that (s)he is still level but just made a mistake in the first large game.

Y \ Z	A (1)	B (2,5)	C (3,4)
a (1,4,5,NE)	0,2	12,20	12,0
b (3,NE)	2,16	18,0	0,20
c (2)	12,6	0,0	8,8

Figure 3: 3x3 G2: the second small game

Y \ Z	A	B (1,2)	C (3,4)
a (1,4,5)	0,0	12,20	12,0
b (2,3)	2,2	18,2	0,20
c (NE)	12,10	0,0	8,0

Figure 4: 3x3 G2: the third small game

Among three small games, the first and third small games possess pure strategy Nash equilibria. Naturally, individuals can reason the structure of games either iteratively or concurrently (i.e., Nash reasoning). As mentioned before, sequential reasoning leads to level-k behaviours. In contrast, concurrent logic is from a game-theoretical perspective and less intuitive, following this type of reasoning, individuals analyze each possible scenario (i.e., what the other player does, including both pure and mixed strategies) independently and construct the best responses for each case. Nash analysis ultimately results in Nash equilibrium behaviour. Remarkably, sequential rationale and concurrent rationale do not necessarily engender the same outcome or choice.

The third small game is designed to identify whether individuals would take distinct behavioural patterns based on particular counterparts. Unlike the 3x3 G1, in which the Nash strategy for player Y can be explained using both level ≥ 2 and Nash reasoning, in this game, the (pure strategy) Nash equilibrium outcome will never be achieved using iterative reasoning. Hence, choosing c is a necessary and sufficient indicator showing the subject is actually using Nash reasoning, provided this player is not choosing randomly (i.e., irrational player).

Last but not least, to eliminate potential order effect and framing effect, the order of three small games, and the order of opponent types are all random in the laboratory implementation. Moreover, the payoffs for player Y and Nash equilibria payoffs are the same for all three small games.

5.2 Part II: Measuring the Valuation

In the second section of the experiment, subjects are asked about their preference between playing the game and sure payment. For each game played against each type of opponent, the subject is firstly asked to confirm their choices in previous section and then instructed to choose between the payoff from this game and each dollar amount from a list between \$8.00 and \$14.00 (25 choices in total with step size \$0.25). Let $\mathcal{L}$ denote the sequence of monetary values one subject chooses against, which is a strictly increasing list. In each game-money choice pair (called a **binary choice problem**) $(g_p, x)\ \forall g \in \mathcal{G},\ \forall p \in \{\text{UG}, \text{Ph.D.}\},\ x \in \mathcal{L}$, let A denote the action of choosing payment from playing game g against opponent of type p, and let B denote the action of choosing the sure payment.

$$\mathcal{L} := \{8 + 0.25k : k \in \{0, 1, \cdots 24\}\} \tag{8}$$

For each g_p, each player's action can be represented using a list with length 25, $\ell_i(g_p) \in \{A, B\}^{25}$. The **switch point** in $\ell_i(g_p)$ is defined to be the index $j \in \{1, 2, \cdots, 25\}$ such that $\ell_i(g_p)_j \neq \ell_i(g_p)_{j-1}$. In the laboratory, it is possible for subjects to show multiple switch points (e.g. jump between A and B) or no switch point (e.g. stick with one choice all the time). However, it can be shown that a player should have at most one switch point if his/her preference is *well-behaved.*

Proposition 5.1. Assuming subject i's underlying preference $\succsim_i$ is both transitive and monotonic (in the space of monetary payoff), there should be at most one switch point in the observed $\ell_i(g_p)$.

Proof. Suppose, for contradiction, there are more than one switch points in $\ell_i(g_p)$, say m and n.

Without loss of generality,

$$\ell_i(g_p) = (A, A, \cdots, A, \ell_i(g_p)_m = B, B, \cdots, B, \ell_i(g_p)_n = A, A, A) \tag{9}$$

Let μ_i denote the belief of player i, then

$$\mathcal{L}_m \succsim_i \mathbb{E}[g_p|\mu_i] \tag{10}$$
$$\mathbb{E}[g_p|\mu_i] \succsim_i \mathcal{L}_n \tag{11}$$

where $\mathbb{E}[g_p|\mu_i]$ represents the subject's expected payoff from game g_p.
By monotonicity, $\mathcal{L}_n > \mathcal{L}_m \implies \mathcal{L}_n \succ_i \mathcal{L}_m$. If $\succsim_i$ is transitive, $\succ_i$ is transitive as well, then

$$\mathbb{E}[g_p|\mu_i] \succsim_i \mathcal{L}_n \succ_i \mathcal{L}_m \succsim_i \mathbb{E}[g_p|\mu_i] \tag{12}$$
$$\implies \mathbb{E}[g_p|\mu_i] \succsim_i \mathbb{E}[g_p|\mu_i] \tag{13}$$

This leads to a contradiction. ■

Proposition 5.2. At any switch point s, the player must switch from action A to action B. That is,

$$\ell_j = A \; \forall j < s \tag{14}$$
$$\ell_j = B \; \forall j \geq s \tag{15}$$

Proof. Suppose, for contradiction, the a player switched from B to A at s for game g_p. Then,

$$\ell_i(g_p)_{s-1} = B \implies \mathcal{L}_{s-1} \succsim_i \mathbb{E}[g_p|\mu_i] \tag{16}$$
$$\ell_i(g_p)_s = A \implies \mathbb{E}[g_p|\mu_i] \succsim_i \mathcal{L}_s \tag{17}$$

The combined results implies

$$\mathcal{L}_{s-1} \succsim_i \mathbb{E}[g_p|\mu_i] \succsim_i \mathcal{L}_s \tag{18}$$
$$\implies \mathcal{L}_{s-1} \succsim_i \mathcal{L}_s \tag{19}$$

However, monotonicity requires $\mathcal{L}_s \succ_i \mathcal{L}_{s-1}$, which is a contradiction. ■

With the hope of analyzing subjects with well-behaved preferences, subjects who violated the transitivity and monotonicity assumptions are discarded.

In this case, sequence of choices measures one subject's value or the expected payoff from a certain game. For example, if player switched from A to B at s in game g_p, then

$$\mathcal{L}_{s+1} \succ_i \mathcal{L}_s \succsim_i \mathbb{E}[g_p|\mu_i] \succsim_i \mathcal{L}_{s-1} \succ_i \mathcal{L}_{s-2} \quad (\dagger) \tag{20}$$

The chain in (†) provides pairs of upper and lower bounds (for both weak preferences) for $\mathbb{E}[g_p|\mu_i]$.

6 Data and Results

6.1 Experimental Data

The studies experiment contains 99 observations from five different sessions, and 323 features characterizing individuals' identities and behaviour patterns. Not all features are relevant to our study, 85 columns are dispensed.

feature name	num.	meaning	possible values
code	1	subject ID	unique values
payoff	1	payoff after experiment	$[0, 24]$
order of opponent (for both sections)	2	whether the subject was playing against undergraduate firstly	{`True`, `False`}
total quiz wrong (part1)	1	number of quizzes did wrong	$\{0, 1, \cdots, 14\}$
all quizzes payoff	1	the total payoff from quizzes	$\{0.5, \cdots, 3.75\}$
decision for game $g \in \mathcal{G}$ and type $p \in$ {undergrad, Ph.D.})	4×2	action taken when played g against p	$\{a, b, c, d\}$
confirmed decision for game $g \in \mathcal{G}$ and type $p \in$ {undergrad, Ph.D.})	4×2	action taken when played g against p confirmed value in section II	$\{a, b, c, d\}$
confirmation difference	4×2	whether the choice in section I is the same as the confirmed in section II	{`True`, `False`}
player's choice for all g and p	$4 \times 2 \times 25$	player's choice in section II	$\{A, B\}$
switching point for all g and p	4×2	player's switching point in section II	$\{0, 1, \cdots, 26\}$
total columns	238		

Table 1: features kept for analysis

In each round of section II, subjects were asked to confirm their choices in previous section. However, because subjects were not exposed to more information of their opponents, in principle, they should not change their choices. A small proportion of subjects changed their mind in this step.

game	opponent type	proportion changed action
4x4	Undergraduate	15(15.1%)
	Ph.D.	20(20.2%)
3x3 G1	Undergradaute	13 (13.1%)
	Ph.D.	12(12.1%)
3x3 G2	Undergradaute	22 (22.2%)
	Ph.D.	17(17.1%)
3x3 G3	Undergradaute	15 (15.1%)
	Ph.D.	21(21.2%)

Table 2: percentage of subjects changed action at the confirmation stage

6.2 Inferring Level of Rationality

Using the criterion of hard levels k, the majority of subjects failed to meet the criterion for any positive level. This is expected as the raw definition of hard levels is pretty strict.

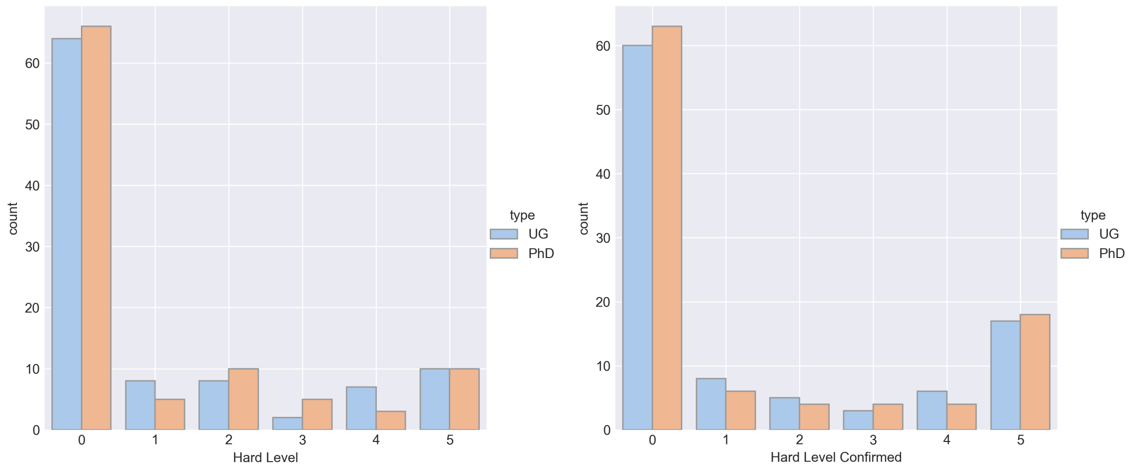

Figure 5: hard levels on original choices and confirmed choices

While allowing at most one mistake in the four rounds (controlling opponent type), more subjects are classified to be higher levels but most subjects are concluded to be level 5 or higher. As mentioned before, the observational data gathered cannot detect these higher level behaviours. This suggests the soft level is too lossy.

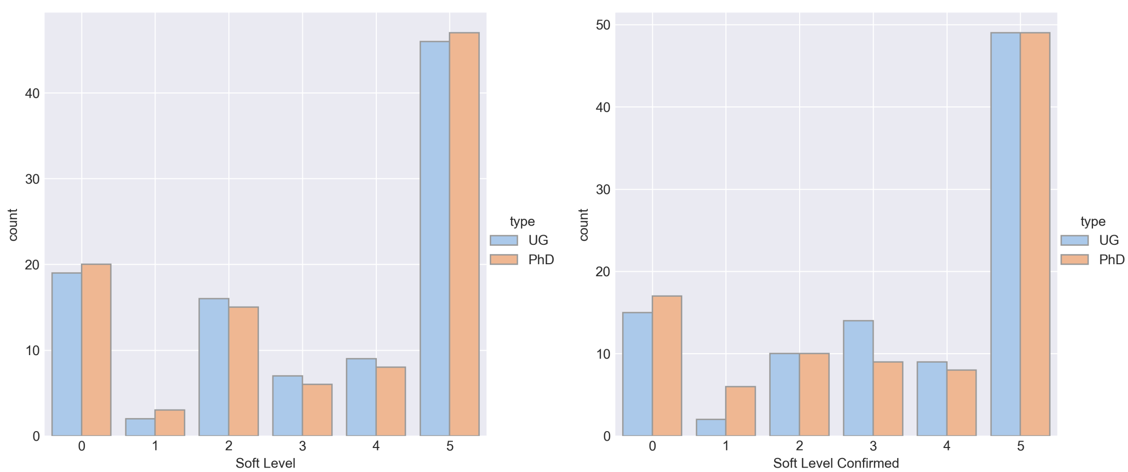

Figure 6: soft levels on original choices and confirmed choices

The controlled soft level criterion is effectively a mixture of the original soft and hard criterion, the rationality levels inferred from confirmed choices distributed evenly across all levels. This even distribution allows more meaningful comparative statics and analysis of impact of opponent types.

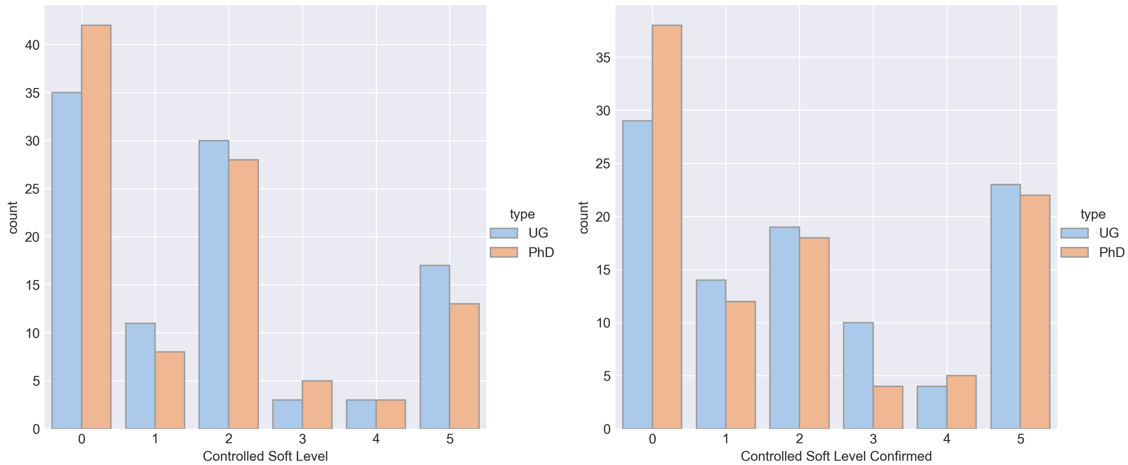

Figure 7: controlled soft levels on original choices and confirmed choices

6.3 Treatment Effect and Level Shifting

The original conjecture of this study is that subjects may switch their levels of rationality while facing opponents of different types. It turns out that, using all three criterions, the levels inferred of around half of subjects changed across sessions. The results are consistent on the dataset of confirmed choices as well. The results suggest that

Observation 6.1. Using all three criterions, for a significant portion of subjects, the inferred levels of rationality switched based on the type of opponent.

criterion	choice	percentage changed
hard	raw	46.4%
	confirmed	44.4%
soft3	raw	57.6%
	confirmed	55.6%
soft3 controlled	raw	55.6%
	confirmed	49.5%

Table 3: proportion of subjects who shifted levels in each setting

However, mixed results are depicted by analyzing the percentage of subjects who jump to a higher level of rationality while facing a Ph.D. opponent. Among who switched levels, around half of subjects showed higher levels while facing a Ph.D. but the rest showed lower levels.

criterion	choice	percentage jumping to a higher level
hard	raw	52.2%
	confirmed	52.3%
soft3	raw	45.6%
	confirmed	51.0%
soft3 controlled	raw	44.9%
	confirmed	44.9%

Table 4: among subjects switched levels, proportion of them jumped to a higher level while facing Ph.D. opponents

Observation 6.2. Among who changed behavioural pattern while facing opponents of different types, the observational data is inconclusive on how exactly these subjects response (i.e. shifts to a higher or a lower level of rationality).

The figure below presents the magnitudes of level shifting measured using ”soft3 controlled” criterion, most changes are clustered within $[-2, 2]$.

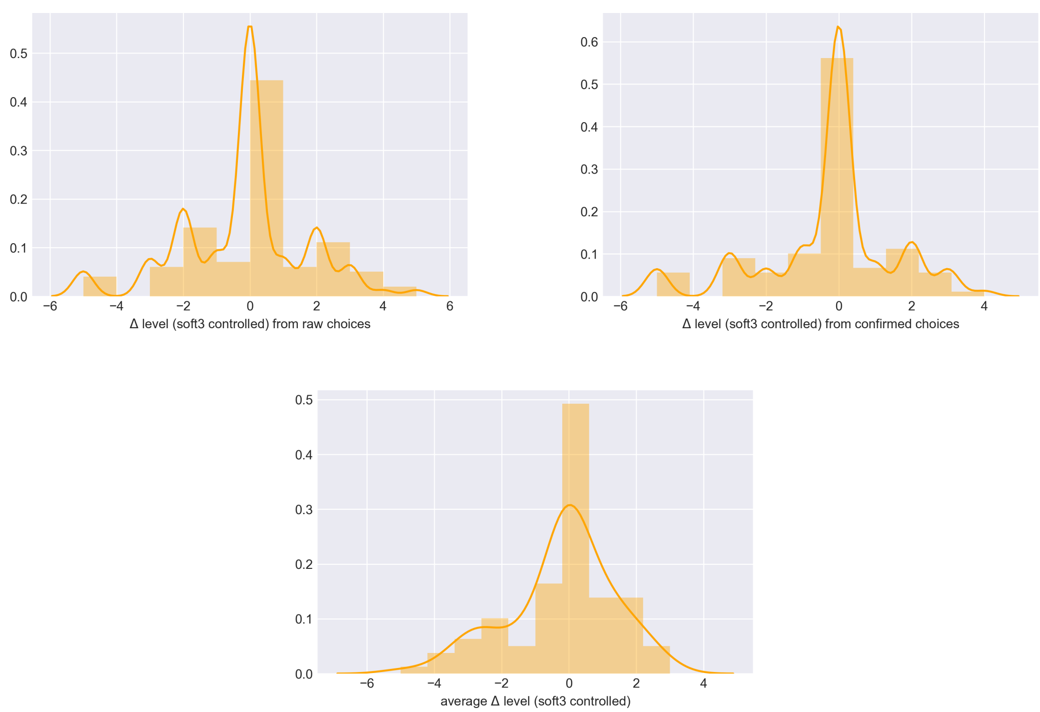

Figure 8: magnitudes of level shifting

6.4 Detecting Nash Reasoning

As mentioned previously, agents, especially those with prior knowledge on game theory, tend to act according to Nash reasoning instead of the iterative mechanics. It turned out that actions of most subjects cannot be rationalized by neither Nash or soft Nash reasoning. While facing undergraduate counterparts, only 12 and 6 subjects were using soft and hard Nash reasoning respectively. When the opponent was switched to Ph.D. students, the numbers increased to 12 and 7. Therefore,

Observation 6.3. Only an insignificant portion of subjects were exerting (soft) Nash reasoning.

6.5 Expected Payoffs from Games

Because of an error in the laboratory implementation, the data on switch points on the 4-by-4 game played against Ph.D. students are missing. The summary statistics of subjects' switching points in game settings other than 4x4 (Ph.D.) session are presented in the table below.

game	opponent type	num. obs.	mean	std.	$\mu_{\text{phd}} - \mu_{\text{ug}}$
4x4	Undergraduate	95	16.22	8.11	
	Ph.D.	3	n/a	n/a	n/a
3x3 G1	Undergraduate	93	11.21	7.18	
	Ph.D.	93	10.57	7.31	-0.64
3x3 G2	Undergraduate	94	10.91	7.48	
	Ph.D.	92	10.84	8.19	-0.07
3x3 G3	Undergradaute	95	13.03	8.34	
	Ph.D.	97	13.07	8.33	-0.01

Table 5: summary statistic for observed switching rows from part two

It seems like subjects would value the same game less if his/her opponent is a Ph.D. student. However, consider the difference is not significant compared with the large standard deviation, the difference remains insignificant.

Observation 6.4. Subjects valued game played against sophisticated opponents less, but the difference is insignificant.

The figure below plots the distribution of switch points for the 3x3 game played against undergraduate, the distributions are similar in other game settings. The histogram suggests the preliminary results are not robust since a great percentage of subjects simply sticked with one action (so the switch point is one) all the time.

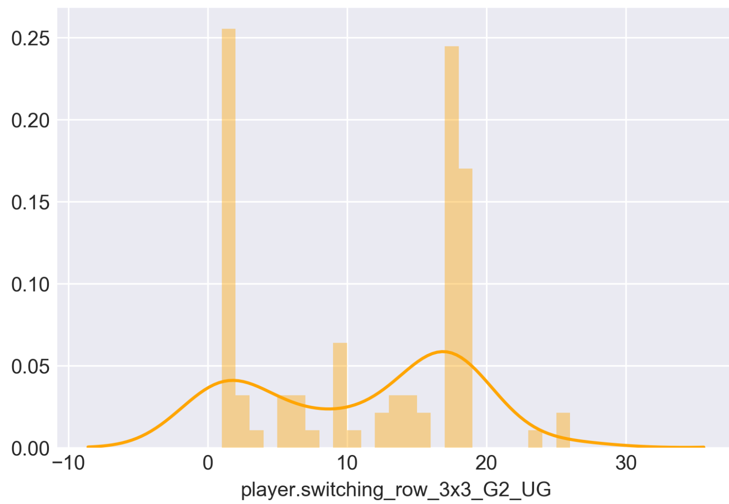

Figure 9: distribution of switch points for 3x3 G2 Undergraduate

To improve the robustness of analysis, the following analysis is restricted to subjects who have switch points strictly greater than 1 and less than 26.

Definition 6.1. A player is said to have **trivial switch point** if this player sticked with one action in all binary choice problems. In this case, his/her switch point is recorded as 1 or 26.

For example, wile looking at the 3x3 G1 game, only those with non-trivial switch point in both 3x3 G1 (UG) and 3x3 G1 (Ph.D.) are considered. The table below shows a summary on these observations.

game	opponent type	num. obs.	mean	std.	$\mu_{\text{phd}} - \mu_{\text{ug}}$
3x3 G1	Undergraduate	58	15.31	3.77	
	Ph.D.	58	14.40	4.91	-0.91
3x3 G2	Undergraduate	51	14.31	4.88	
	Ph.D.	51	13.59	5.40	-0.72
3x3 G3	Undergradaute	49	15.82	4.26	
	Ph.D.	49	15.29	4.28	-0.53

Table 6: summary statistic for observed switching rows from part two (after filtering, 4x4 games excluded)

The magnitudes of differences $\mu_{\text{phd}} - \mu_{\text{ug}}$ are more profound compared with results from the entire dataset, moreover, the standard deviations are almost halved (due to the reduced number of observation). Therefore,

Observation 6.5. After restricting to subjects who had non-trivial switch point. The level shifting behaviour is more profound, and magnitudes of level shifting are more significant.

7 Discussions and Concluding Remarks

The experiment is designed to capture how rational and sophisticated players behave while facing opponents at different levels. Further, the observational data provides evidence showing a significant proportion of subjects acted at different levels while playing against undergraduates and Ph.D. candidates. However, the direction of change remains unclear. The second section of experiment measures subjects' expected payoff from each game. Subjects did, on average, expect less from games played against Ph.D. opponents, but the magnitude of difference is less salient compared with the standard deviation.

Due to the limited space, many aspects of potential improvements are not discussed in full details in this paper. One consideration is the definition for level 0 behaviour. The definition of level 0 rationality is the starting point of the entire model, and further analysis is sensitive to this initial condition. There are several alternative definitions of level 0: for instance, level 0 players can be assumed to always choose the action associated with highest possible payoff. Another possible assumption on level 0 is that the player is always choosing the action with the greatest lower bound on corresponding payoffs.

References

Alaoui, Larbi, and Antonio Penta. 2016. Endogenous depth of reasoning. *Review of Economic Studies* 83 (4): 1297–1333. ISSN: 1467937X. doi:10.1093/restud/rdv052.

Kneeland, Terri. 2015. Identifying Higher-Order Rationality. *Econometrica* 83 (5): 2065–2079. ISSN: 0012-9682. doi:10.3982/ecta11983.

Nagel, Rosemarie. 1995. Unraveling in Guessing Games: An Experimental Study. *American Economic Review* 85 (5): 1313–1326. doi:10.2307/2950991.