Information Release during COVID-2019

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1 Introduction

1.1 Background

During the time of COVID-2019, governments in affected countries are faced with challenges they have rarely handled before. If the disease outbreaks in one city, what should the government do to keep the spreading of disease under control? In particular, the Centre for Disease Control and Prevention (CDC) has to inform the public about the epidemic situation, including numbers of probable cases, confirmed cases, and the fatality. Since the first case of COVID-2019 infections was reported in last December, many people have been skeptical about these numbers announced by the government. Sources are suspecting the number of infections has been under-reported, both intentionally and unintentionally (Gan 2020, Klebnikov 2020, Cadell 2020, Fifield 2020).

This paper aims to examine whether under-reporting the severity of the disease is rational for government using game-theoretical frameworks. Truthful reporting helps the general public to better prepare for the upcoming epidemic situation and help slow down the spread of disease. However, when the situation is terrifying, real information can cause public panic.

1.2 Strategic Environment

1.2.1 Government

The government aims to control the spread of disease and minimizes the total cost of treatment. The government has the access to all hospital records and, therefore, the actual epidemic situation. Everyday, government has to announce the epidemic situation to residents in the city. While releasing information, the government has to decide *whether to report the number of infections within its area of administration truthfully*. With the actual number of confirmed cases, government can compute the actual number of infections as a percentage of city population. For simplicity, we define this percentage to be the actual situation (severity) of disease:

$$\text{Actual Situation} := \frac{\text{Actual Number of Confirmed Cases}}{\text{City Population}} \in [0, 1] \tag{1.1}$$

Without loss of generality, this papers assumes that the government is announcing the number of confirmed cases in terms of percentage of population every day. For example, suppose there are ten thousand ac-

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cumulated confirmed cases in a city with 1 million population, then the actual situation of disease is 1%, and government can under-report the severity by announcing 0.5%, over-report the situation by announcing 3.0%, or report truthfully. In general, government can report any percentage between 0% and 100%.

There are several ways for the government to misreport the actual situation legally. One method is to use an alternative definition of confirmed cases. Before February 11, 2020, the daily growth of confirmed cases was on a hundred basis in mainland China. However, the number of confirmed cases increased by 15,000 on February 11. This spike far exceeded the growth rate of an exponential growth. The discontinuity happened because the Chinese government changed to a broader definition of being infected. In essence, the suspected patient was confirmed before only if the nucleic acid test gives a positive result, the border criterion classifies a patient to be infected if she shows typical symptoms. Similarly, government can under-report the severity by using a stricter definition of being infected and a broader criterion of being recovered.

1.2.2 Residents

Residents care about their health and property, but during the incubation period, one will not know if she is infected. Therefore, the following analysis is the same for both infected (not tested/confirmed yet) and healthy people. Due to the lack of knowledge about epidemiology and limited access to information, residents do not know the actual epidemic situation. All residents know before the government announcement is the actual situation follows their prior belief. In most cases, residents' prior belief is uniform due to the lack of information. After the government announces the number of infections (again, not necessarily to be the real situation), households form a posterior belief about the epidemic situation. Based on this posterior belief, the cost of treatment, the cost of evacuation, residents have to decide whether to stay in the city.

For instance, suppose the actual situation is severe and the likelihood of being infected (as well as the expected cost of treatment) is high for residents who decided to stay. Consider someone with a personal vehicle and is able to work remotely, the cost of evacuating is relatively low for her. In this case, her best response would be running away to other cities where no infection has been found. In contrast, for someone with prohibitively high cost for evacuation, she would be better off staying in the city.

1.3 Strategic Behaviour and Payoffs

Assumption 1.1. In this model, we assume that the total cost of treatment to be distributed between the patient and the government via a health care insurance system. Moreover, we assume the total cost to be the same for all patients, and all patients pay the same percentage of medical cost, say 50%.[1]

In this framework, residents' payoffs are simply the negation of their expected expenses. For residents decided to stay in the city, their expected expense is the cost of medical treatment (total cost of treatment minus the portion covered by national medical insurance) multiplied by the likelihood of being infected while staying in the city.

$$\text{Payoff(Stay)} = -\text{Cost of Treatment} \times \text{Likelihood of Infection} \tag{1.2}$$

Similarly, the cost of running away is the transportation cost plus forfeited income, if working remotely is impossible for this person. For simplicity, we do not distinguish these two sources of cost in subsequent discussions. Hence, the cost of evacuation mentioned later denotes the aggregate cost.

The cost of medical treatment is distributed evenly to both government and patients through some medical care insurance system. Therefore, the major part of government's expense is the cost of treatment (total treatment cost minus the portion paid by patients). Because the virus is highly contagious, patients

[1] The exact percentage of medical cost covered by insurances does not really matter in this model.

need to be hospitalized or isolated immediately after being tested positive for infection. It is pretty rare for patients tested positive in the city to be transferred to hospitals in another city, vice versa. Hence, government's cost is the sum of costs for in-city patients and out-of-city patients, and government's payoff is precisely the negative cost.

$$\text{Government's Payoff} = -C(\text{Number of Patients in the City}) - C(\text{Number of Patients outside the City}) \tag{1.3}$$

Where C is a cost function with increasing marginal cost, which attributes to the scarcity of local medical resources such as ICU beds.

After government announces the epidemic situation and residents analyze the cost of their available options (i.e., staying or running away), a proportion of residents would decide to evacuate. Among these people who run away from the city, a proportion of them has already been infected. As a result, the number of patients in the city decreases. However, these infected people running away spread the disease to other cities, and the number of patients outside the city would increase much more. Note that for every infected person who evacuate from the city, they spread the virus and the total number of infections will rise.

By announcing a lower number than the actual number of infections, government encourages more residents, hence, patients, to stay in the city. Under-reporting helps prevent the disease from spreading but increases the pressure on local healthcare facilities, and the cost of treating one additional patient in the city rises as more patients are hospitalized. In contrast, if government announces a higher number of infections, more patients are running away to other cities and later tested and treated in other cities. This helps alleviate the pressure on local healthcare system but worsens the overall epidemic situation (i.e., the total number of patients, in and out of the city).

2 Models

2.1 A Strategic Form Game with Incomplete Information

2.1.1 Model

The baseline model proposed is a strategic form game with discrete types and actions. In this minimal model, there is only one household in the city. The payoffs of the government and residents depend on the actual situation $\omega^* \in \{\omega_\ell, \omega_h\}$. Given the situation to be good (ω_ℓ, i.e., low number of infections) or bad (ω_h, i.e., high number of infections), government can choose to report low (b_ℓ) or high (b_h). As the response to government's announcement, residents make decisions to evacuate from the city or stay.

When the situation is good, if residents choose to stay in the city, there's no difference between government's payoff from two actions. All the government needs to pay is the cost of medical treatments for the small number of patients (5). If residents choose to evacuate, an announcement of high number of infections would worsen the public panic and results in a relative higher cost (50 v.s. 20). As for residents, the cost of evacuating from the city is the transportation cost (10). When the situation is mild, the expected cost from staying is negligible (0).

ω_ℓ	Run Away	Stay
b_h	(-50, -10)	(-5, 0)
b_ℓ	(-20, -10)	(-5, 0)

Table 1: Payoffs in ω_ℓ Good/Mild Situation

When the situation is difficult, the government's payoff depends on residents' actions only and does not depend on the announcement anymore. Because of the scarcity of medical resources in the city, the government is indeed better off when residents choose to run away (so that patients running away may use hospitals in nearby cities). As for residents, the cost of evacuating is higher (15) because of factors including the increased volume of passengers running away, possible traffic control by the government (e.g., lockdown), and risk of being infected during the trip. Therefore, the expected cost of staying is substantially higher due to the increased risk of infections and shortages of medical resources in the city.

ω_h	Run Away	Stay
b_h	(-70, -15)	(-100, -50)
b_ℓ	(-70, -15)	(-100, -50)

Table 2: Payoffs in ω_h Bad Situation

2.1.2 Equilibrium

Suppose that residents are holding the prior belief of the severity

$$\mu(\omega_h) = p \quad \mu(\omega_\ell) = 1 - p \tag{2.1}$$

Residents understand that, in order to avoid unnecessary public panic, their government is better off by reporting b_ℓ when the situation is mild. However, they do not know; hence, residents' beliefs on government's actions are

$$\mu(b_\ell|\omega_\ell) = 1 \quad \mu(b_h|\omega_\ell) = 0 \tag{2.2}$$

$$\mu(b_\ell|\omega_h) = q \quad \mu(b_h|\omega_h) = 1 - q \tag{2.3}$$

Using Bayes rule, residents can update their posterior beliefs of the actual situation conditioned on government's action.

$$\mu(\omega_\ell|b_\ell) = \frac{\mu(b_\ell|\omega_\ell)\mu(\omega_\ell)}{\mu(b_\ell)} = \frac{\mu(b_\ell|\omega_\ell)\mu(\omega_\ell)}{\mu(b_\ell|\omega_\ell)\mu(\omega_\ell) + \mu(b_\ell|\omega_h)\mu(\omega_h)} = \frac{1-p}{1-p+qp} \tag{2.4}$$

$$\mu(\omega_h|b_\ell) = 1 - \mu(\omega_\ell|b_\ell) = \frac{pq}{1-p+pq} \tag{2.5}$$

The probability in equation (2.5) is the likelihood that government conceals the actual situation when the government reports b_ℓ (i.e., underreporting the epidemic situation).

Since residents know that reporting b_ℓ is a dominant strategy in situation ω_ℓ, they can deduce that the situation must be bad when government reports b_h:

$$\mu(\omega_\ell|b_h) = \frac{\mu(b_h|w_\ell)\mu(\omega_\ell)}{\mu(b_h)} = \frac{0\mu(\omega_\ell)}{\mu(b_h|\omega_h)\mu(\omega_h) + \mu(b_h|\omega_\ell)\mu(\omega_\ell)} = 0 \tag{2.6}$$

$$\mu(\omega_h|b_h) = 1 \tag{2.7}$$

While deciding whether to evacuate or stay, residents maximize their expected payoffs. Let $U_R(a|b)$ denote the expected payoff from choosing $a \in \{\text{Run Away}, \text{Stay}\}$ after government announces $b \in \{b_\ell, b_h\}$, then

$$U_R(\text{Run Away}|b_\ell) = \mu(\omega_h|b_\ell)(-15) + \mu(\omega_\ell|b_\ell)(-10) \tag{2.8}$$
$$= -15\frac{pq}{1-p+pq} - 10\frac{1-p}{1-p+pq} \tag{2.9}$$
$$= \frac{5-5p}{1-p+pq} - 15 \tag{2.10}$$
$$U_R(\text{Stay}|b_\ell) = \mu(\omega_h|b_\ell)(-50) + \mu(\omega_\ell|b_\ell)(0) \tag{2.11}$$
$$= -50\frac{pq}{1-p+pq} \tag{2.12}$$
$$U_R(\text{Run Away}|b_h) = -15 \tag{2.13}$$
$$U_R(\text{Stay}|b_h) = -50 \tag{2.14}$$

Because of equation (2.2), citizens know that their government will never under-report the situation, so running away is always the best response for residents when government announces b_h. When government is announcing b_ℓ, run away if and only if:

$$U_R(\text{Run Away}|b_\ell) \geq U_R(\text{Stay}|b_\ell) \tag{2.15}$$
$$\iff \frac{9(p-1)}{1-p+pq} + 7 \geq 0 \tag{2.16}$$

Assuming residents do not have any prior knowledge of the epidemic situation so that their prior belief on ω is uniform. Condition (2.16) can be further simplified by plugging in $\mu(\omega_h) = p = \frac{1}{2}$.

$$(2.16) \iff \frac{9}{q+1} \leq 7 \iff \mu(b_\ell|\omega_h) \geq \frac{2}{7} \tag{2.17}$$

2.1.3 Interpretation of this Equilibrium

When the situation is mild, the government is better off reporting low in order to prevent the public from being panic. However, we assume that when the situation is out of control, there is no difference for the government to report high and low. Type ω_h government can report either b_ℓ or b_h.

After government announces b_h, residents are better off evacuating from the city since they know the actual situation must be ω_h. Residents' behaviours after hearing an announcement of b_ℓ are more interesting, the best response depends on how trust-worthy residents think the government is. If the public does not trust their government and believe their government might under-report the situation with a high likelihood (in particular, when (2.17) holds), they are still going to evacuate even the government announces b_ℓ.

2.2 An Extensive Form Game with Incomplete Information

The baseline strategic form game provides insights on residents' responses to government's announcement. However, in reality, residents are deciding after hearing government's announcement. Therefore, this scenario can be better modelled using an extensive form game instead of letting agents act simultaneously.

The extensive form game consists of one government G and N residents $\{R_1, R_2, \cdots, R_N\}$, where N is large. Further, let h_G and h_{R_i} denote the private history of government and resident i respectively.

2.2.1 Stage 0: The State of World

Let the continuous variable $\omega^* \in \Omega \equiv [0, 1]$ denote the **actual epidemic situation**, that is, the state of the world. One interpretation of ω^* can be the proportion of population who are infected. In this case,

$\omega^* = 0$ denotes the best situation (i.e., nobody in the country is infected). In contrast, $\omega^* = 1$ denotes the worst situation (i.e., everyone is infected). For simplicity, we normalize the whole population to one unit, so that ω^* represents both the proportion and the number of infections. This paper assumes the government has access to all hospital records and reports from epidemiology experts so that the government has full information on the real situation.

Assumption 2.1. The government observes the actual epidemic situation ω^*.

Assumption 2.2. Residents do not observe the actual epidemic situation.

After the realization of the state of world, the private history of government, h_G, is extended to $h_G = (\omega^*)$. Since only government observes the state of world, residents' private histories h_{R_i} remain empty.

$$h_G = (\omega^*) \tag{2.18}$$

$$h_{R_i} = \varnothing \quad \forall i \in \{1, 2, \cdots, N\} \tag{2.19}$$

2.2.2 Stage 1: Government's Announcement

After each history $h_G = (\omega^*)$, the government has to choose a number, $b \in A_G = \Omega$, in its daily information release to inform the public how severe the situation is. Government does not necessarily unravel the true situation to the public if the consequence of public panic is prohibitive. Since government only acts once in this game, therefore, a strategy for the government is a mapping from Ω to $A_G = \Omega$.

$$\sigma_G : \Omega \to A_G \tag{2.20}$$

As mentioned in the previous section, when the epidemic situation is relatively mild, the government can keep citizens calm and prevent the virus from spreading to other cities by under-reporting the situation. In contrast, when the situation is severe, the government is better off reporting the actual number of infections to make sure citizens are protecting themselves.

Equation (2.21) gives the government's payoff as a function of the actual number of infections.

$$u_G(\omega, \pi, \theta) = -(\omega - \omega\pi)^2 - (\pi(1 + \omega)\theta)^2 \tag{2.21}$$

After the government makes its announcement regarding the epidemic situation, a proportion, $\pi \in [0, 1]$, of residents in the city become panic and choose to evacuate to other cities. Hence $\omega\pi$ denotes the number of infected people who run away after the announcement. θ is the branching factor measuring the transmission potential of the disease. In particular, $\theta - 1$ is the average number of secondary infections produced by a typical patient in a population where everyone is susceptible. [2] As the situation is getting worse, the likelihood of the virus to mutate and become more contagious is increasing. To capture this feature, we use $(1+\omega)\theta$ to denote the actual branching factor. The actual branching factor is an increasing function in both epidemic situation ω and the base branching factor θ. Therefore, θ counts the total number of infected cases induced by one patient.

Since there is no vaccines at the beginning of a novel disease outbreak, patients would have to stay in hospitals much longer compared with normal diseases. The scarcity of medical resources (e.g., hospital beds and ICU units) causes great difficulty in finding hospital beds for patients diagnosed later. Therefore, this model assumes a quadratic cost of treating patients with increasing marginal costs. In general, patients in the city are hospitalized in the city; there is no competition for medical resources between patients within and out of the city, the aggregate cost is the sum of in-city cost and out-of-city cost. $\omega - \omega\pi$ and $\pi\omega\theta$

[2] The number $\theta - 1$ is called the **reproduction number** in epidemiology literature, and it is often written as R_0.

are numbers of infections inside and outside the city. So that equation (2.21) measures the negative cost of treating all in-city and out-of-city patients. The proportion of residents running away is an increasing function of b, so that government's problem can be expressed as

$$\max_{b\in[0,1]} \mathbb{E}[u_G(\omega, \pi(b), \theta)] \tag{2.22}$$

and the optimal number to be announced given true epidemic situation ω^* is

$$\beta^*(\omega^*) = \underset{b\in[0,1]}{\operatorname{argmax}} \mathbb{E}[u_G(\omega, \pi(b), \theta)] \tag{2.23}$$

Clearly, β^* depends on the government's expectation of public responsiveness, $\pi(b)$.

After the government announces b, the private histories are now

$$h_G = (\omega^*, b) \tag{2.24}$$

$$h_{R_i} = (b) \quad \forall i \in \{1, 2, \cdots, N\} \tag{2.25}$$

2.2.3 Stage 2: Residents' Responses

After history $h_{R_i} = (b)$, that is, hearing government's announcement regarding the epidemic situation, every resident i has to choose an action from $A_{R_i} = \{\text{Run Away}, \text{ Stay}\}$. Therefore, one valid strategy for household i is a mapping from government's announcement to an action in A_{R_i}.

$$\sigma_{R_i}(b) \to A_{R_i} \tag{2.26}$$

The final decision of one resident depends on her posterior belief on the epidemic situation as well as her costs of transportation and medical treatment if she got infected.

Residents in the city, in general, do not have much experience on diseases, they hold a uniform **prior belief** of ω^*, that is,

$$\omega \sim \text{Unif}(0, 1) \tag{2.27}$$

$$\mu_i(\omega) = 1 \ \forall \omega \in [0, 1] \tag{2.28}$$

After learning the number of infections announced by the government, $b \in [0, 1]$, through the public announcement, residents evaluate their **posterior belief**:

$$\mu_i(\omega|b) \tag{2.29}$$

and make decisions according to this posterior belief. As mentioned before, residents have two options after hearing the public announcement: staying in the city and running away.

Let C denote the cost of medical treatment and assume the virus is super contagious, it is reasonable to assume the likelihood for someone staying in the city to be infected is proportional to the current number of infection. For simplicity, we assume the likelihood of infection to be exactly the percentage of people infected, ω. Then the expected payoff for staying in the city is the likelihood of infection multiplied by the cost of treatment:

$$u_R(\text{Stay}) = -\omega C_i \tag{2.30}$$

Meanwhile, the cost of running away consists of both transportation cost and the cost of being infected during the trip. Specifically, let T_i denote resident i's cost of running away. [3]

Assumption 2.3. We assume the transportation cost to be lower than the cost of medical treatment, $T_i < C_i$.

Let $\rho(\omega)$ be the probability (risk) of being infected during the trip and outside of the city. If it is risker to runaway than staying in the city, staying would be a dominant strategy for residents, and the equilibrium solution would be trivial. This model assumes it is relatively safer, in terms of likelihood of being infected, to evacuate from the city:

$$\forall \omega \in \Omega,\ 0 \leq \rho(\omega) \leq \omega \tag{2.31}$$

For now, assume $\rho(\omega) = 0$ for all ω, that is, guaranteed safety from virus outside the city. Therefore, the cost of evacuating from the city is simply T_i:

$$u_R(\text{Run Away}) = -T_i - \rho(\omega)C_i = -T_i \tag{2.32}$$

As we will show later, only the ratio of T_i and C_i matters for residents' behaviours. Therefore, we may define the type of resident i to be the ratio of T_i and C_i.

$$t_i = \frac{T_i}{C_i} \tag{2.33}$$

Because residents do not observe the true state ω^* directly, they have to form a posterior belief of ω^* based on government's announcement b, denoted as $\mu_i(\omega^*|b)$. The expected payoffs based on residents' poster beliefs are

$$U_R(\text{Stay}) = -\mathop{\mathbb{E}}_{\omega\sim\mu_i(\omega|b)}[\omega C_i] = -C_i \mathop{\mathbb{E}}_{\omega\sim\mu_i(\omega|b)}[\omega] \tag{2.34}$$

$$U_R(\text{Run Away}) = -T_i \tag{2.35}$$

Residents compare their expected payoffs from these two actions available and act accordingly, someone is better off choosing to evacuate from the city if and only if

$$U_R(\text{Run Away}) \geq U_R(\text{Stay}) \tag{2.36}$$

$$\iff -T_i \geq -C_i \mathop{\mathbb{E}}_{\omega\sim\mu_i(\omega|b)}[\omega] \tag{2.37}$$

$$\iff \frac{T_i}{C_i} \leq \mathop{\mathbb{E}}_{\omega\sim\mu_i(\omega|b)}[\omega] \tag{2.38}$$

$\frac{T_i}{C_i}$ is different for different residents, however, we may impose some assumptions on the distribution of $\{\frac{T_i}{C_i}\}_{i=1}^N$. The ratio $\frac{T_i}{C_i}$ of a randomly chosen citizen i follows some distribution that has density function f with support $[0, 1]$. Since the transportation cost T_i implicitly includes the opportunity cost of not working, most people have relatively high values of $\frac{T_i}{C_i}$.

Assumption 2.4. The relative transportation cost $\frac{T_i}{C_i}$ follows a distribution with density function $f(x) = 2x$.

This formulation suggests that some residents with relatively lower values of $\frac{T_i}{C_i}$ are more likely to run away after hearing the public announcement. The proportion of population decides to evacuate is exactly the value of π in government's payoff (2.21).

[3] As mentioned in the first section, the transportation cost implicitly includes the loss of income when working remotely is impossible.

3 Results for the Extensive Form Game

3.1 Equilibrium

An equilibrium can be derived using backward induction. Like what is happening in US after the COVID-19 outbreak, the majority of population are suspecting that the government is threatening them by exaggerating the severity of disease. This is another key point of how the second model differs from the first one.

Assumption 3.1. Because residents do no have any information other than government's report, the posterior belief of the actual situation after hearing someone report of value b is a uniform distribution between $[0, b]$. Hence,

$$\mathop{\mathbb{E}}_{\omega \sim p(\omega|b)}[\omega] = \frac{b}{2} \tag{3.1}$$

Based on the distribution of $\frac{T}{C}$ in assumption (2.4), the proportion of residents running away after receiving history $h_{R_i} = (b)$ satisfies

$$\pi(b) = \int_0^1 f(x)\mathbb{1}\{x \leq \frac{b}{2}\}\ dx = \int_0^{\frac{b}{2}} 2x\ dx = \frac{b^2}{4} \tag{3.2}$$

The government's payoff in equation (2.21) can be written as

$$\begin{aligned} u_G(\omega, \pi(b), \theta) &= -(\omega - \omega\pi(b))^2 - (\pi(b)(1+\omega)\theta)^2 && (3.3)\\ &= -\left(\omega - \omega\frac{b^2}{4}\right)^2 - \left(\frac{b^2}{4}(1+\omega)\theta\right)^2 && (3.4)\\ &= -\frac{1}{16}b^4\theta^2(\omega+1)^2 - \left(1 - \frac{b^2}{4}\right)^2\omega^2 && (3.5) \end{aligned}$$

Government's optimization problem is

$$\max_{b\in[0,1]} -\frac{1}{16}b^4\theta^2(\omega+1)^2 - \left(1 - \frac{b^2}{4}\right)^2\omega^2 \tag{3.6}$$

The government's optimal action parameterized by ω and θ is

$$\beta^*(\omega, \theta) = \mathop{\mathrm{argmax}}_{b\in[0,1]} -\frac{1}{16}b^4\theta^2(\omega+1)^2 - \left(1 - \frac{b^2}{4}\right)^2\omega^2 \tag{3.7}$$

The first-order necessary condition of (3.6) is

$$\begin{aligned} &\frac{\partial}{\partial b}\bigg|_{b=b^*} -\frac{1}{16}b^4\theta^2(\omega+1)^2 - \left(1 - \frac{b^2}{4}\right)^2\omega^2 = 0 && (3.8)\\ &\implies b^*\omega^2 - \frac{1}{4}b^{*3}\left(\theta^2(\omega+1)^2 + \omega^2\right) = 0 && (3.9) \end{aligned}$$

There are three solutions to (3.9):

$$b_1 = 0 \tag{3.10}$$

$$b_2 = -\frac{2\omega}{\sqrt{\theta^2(\omega+1)^2+\omega^2}} \tag{3.11}$$

$$b_3 = \frac{2\omega}{\sqrt{\theta^2(\omega+1)^2+\omega^2}} \tag{3.12}$$

The function u_G is even in b, therefore $u_G(b_1) = u_G(b_2)$. However, since $\omega \geq 0, \theta \geq 1$, b_2 is always negative and is not in the feasible set. Therefore, the third solution (3.12) is the optimal action for government.

Let $\Theta = [1, \infty)$. Note that for any combination of $(\omega, \theta) \in \Omega \times \Theta$, the best response is within the feasible range Ω.

3.2 Equilibrium Strategies and Beliefs

The previous backward induction algorithm implies that the government's optimal strategy and belief can be summarized as

$$\sigma_G(\omega) = \beta^*(\omega, \theta) = \frac{2\omega}{\sqrt{\theta^2(\omega+1)^2+\omega^2}} \tag{3.13}$$

$$\mu_G\left(\frac{T_i}{C_i}\middle|\,\omega\right) \sim \text{Unif}(0,1)\ \forall \omega \in \Omega \tag{3.14}$$

Similarly, resident i's equilibrium strategy and belief are

$$\sigma_{R_i}\left(b\middle|\,t_i = \frac{T_i}{C_i}\right) = \begin{cases} \text{Run Away if } \frac{T_i}{C_i} \leq \frac{b}{2} \\ \text{Stay otherwise} \end{cases} \tag{3.15}$$

$$\mu_i(\omega|b) \sim \text{Unif}(0, b) \tag{3.16}$$

3.3 Comparative Statics

In the extensive form game, government's optimal behaviour depends on how contagious the disease is and the actual number of infections. In particular,

$$\frac{\partial \beta^*(\omega,\theta)}{\partial \omega} = \frac{2\theta^2(\omega+1)}{\left(\theta^2(\omega+1)^2+\omega^2\right)^{3/2}} \geq 0 \tag{3.17}$$

$$\frac{\partial \beta^*(\omega,\theta)}{\partial \theta} = -\frac{2\theta\omega(\omega+1)^2}{\left(\theta^2(\omega+1)^2+\omega^2\right)^{3/2}} \leq 0 \tag{3.18}$$

The positive partial derivative in (3.17) suggests that when the epidemic situation is getting worse, government is always better off reporting a higher number for each level of branching factor θ. Because the cost of in-city patients and out-of-city patients are separated in government's payoff (2.21), re-allocate some patients to hospitals out of the city can help reduce the total cost of treatment. The observation in (3.17) corresponds to the fact that announcing a higher level would encourage more patients to evacuate and relieve the pressure on local medical facilities.

The negative partial derivative in (3.18) implies that the government should report a lower number for every level of ω if the disease becomes more and more contagious. The rationale behind is that the government can reduce the number of patients running away by announcing a lower number of infections.

When the branching factor θ is sufficiently high, the potential impact induced by letting one patient run out of the city is prohibitively high.

For instance, when the virus is not super contagious, say $\theta = 1.5$, government would report truthfully (over-report 4%) when the situation is mild ($\beta^*(0.25, 1.5) = 0.26$). When the situation is severe, say $\omega = 0.75$, government will under-report the situation for around 25% ($\beta^*(0.75, 1.5) = 0.55$).

The story is different when the government is facing a virus with a larger branching factor. Current literatures suggest that COVID-19 has a branching factor $\theta \approx 3.5$ (Hellewell et al. 2020). For this kind of contagious disease, the government tends to under-report the actual situation much more than the previous case. With $\theta = 3.5$ and $\omega = 0.25$ (mild situation), government's optimal action is $\beta^*(0.25, 3.5) = 0.11$, which is 56% lower than the actual ω. When the situation of a contagious virus is severe, government will report an even lower number $\beta^*(0.75, 3.5) = 0.24$, which is a 68% under-reporting.

3.4 Over-reporting Behaviour

One interesting phenomenon is that the government may actually over-report the situation, especially when the number of infection is low, and the branching factor is small. The over-reporting behaviour corresponds to the scenario in which the local medical resources are sufficient, and the government try to encourage residents to stay in the city to prevent spread of the disease to other cities. For instance, when $\theta = 1.5$ and $\omega = 0.2$, government's optimal number to report is 0.22 (10% over-reporting). In particular, government is better off over-reporting whenever

$$\beta^*(\omega, \theta) = \frac{2\omega}{\sqrt{\theta^2(\omega+1)^2 + \omega^2}} \geq \omega \tag{3.19}$$

The inequality holds if $\omega \geq 0 \wedge \omega = 0$ (the trivial case) or

$$1 \leq \theta < 2 \tag{3.20}$$

$$\text{and } 0 \leq \omega \leq \sqrt{\frac{3\theta^2+4}{(\theta^2+1)^2}} - \frac{\theta^2}{\theta^2+1} \tag{3.21}$$

These two conditions suggest that, when the virus is not contagious (condition 3.20) and the percentage of infections is below a certain threshold (condition 3.21), over-reporting is a best response for the government. Above analytical solutions match the intuitive explanation to government's over-reporting behaviour analyzed above.

4 Concluding Remarks

4.1 Policy Implications

This paper focuses on the interaction between government and residents during the period of epidemic disease. Two models in this paper examine whether it is rational for the government to report the epidemic situation truthfully, assuming residents' beliefs and behaviours are simple enough.

The first model focuses on the resident side, the main implication of this model is that residents' behaviours are influenced by both the cost of medical treatment and how trustworthy they believe their government is. In particular, when the public believe their government may spread misinformation (i.e., under-report the situation), it is possible for residents to choose to evacuate even when the government is claiming the situation is mild.

In contrast, the second model describes the interaction as an extensive form game and focuses more on the government side. In the second model, the state of world and government's action are extended to continuous spaces. Moreover, the second model incorporates an additional branching factor parameter to model how contagious the virus is. Intuitively, government has incentives to conceal the actual situation to safeguard the stability of economy. However, as what is observed in the U.S. in March 2020, the general public tends to under-estimate the severity of disease. In fact, many are believing the COVID-19 is just another flu. As this paper has shown in the equilibrium section, the government actually has incentives to over-report the situation especially when the situation is mild and the branching factor is low. By over-reporting the severity, the government can raise public awareness of the disease and potentially relief the pressure on local healthcare facilities. In general, the extensive form game implies government's optimal announcement (β^*) increases in the severity of disease (ω) but decreases in the branching factor (θ). When the branching factor θ is high, government is better off under-reporting the situation and discouraging people from running away and spreading the virus to other cities.

4.2 Limitations and Extensions

In real world, government can take other measures such as lockdown to prevent residents from running away and save the government's reputation at the same time. One limitation of this framework is that government can prevent residents from evacuating by soft actions (e.g., making announcements) only, and then hopes residents choose to stay at home. When both ω and θ are at high levels, the government should exert stronger means to prevent the spread of virus. The action space of government can be extended to

$$A'_G := A_G \cup \{\text{Lock Down}\} \tag{4.1}$$

Moreover, this model does not incorporate government's reputation. The government can spread misinformation (either over-report or under-report the situation) at no cost. In reality, most governments value their public credibilities heavily and mis-reporting the situation harms their credibilities. To incorporate this feature into our model, the government's payoff in equation (2.21) can be extended to

$$u_G(\omega, \pi, \theta, b)' := u_G(\omega, \pi, \theta) - f(|b - \omega|) \tag{4.2}$$

where f is an increasing function. The last term in the new payoff of government,$f(|b-\omega|)$, measures the penalty of spreading misinformation.

There are potential improvements to the residents' behaviours as well. It is reasonable to assume residents' prior beliefs to be uniform distributions, but assuming their posterior belief to be truncated uniform distributions (Assumption 3.1) can be an oversimplification. One alternative posterior is truncated Gaussian distributions

$$\mu_{R_i}(\omega|b)' := \frac{\phi(\omega)}{\int_0^1 \phi(x)\ dx} \tag{4.3}$$

where ϕ is the density function of $\mathcal{N}(b, \sigma^2)$.

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